Two-Dimensional, Laminar, Compressible Boundary-Layer Calculations in Turbomachines

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Nomenclature

c_f = local friction coefficient
r = radius, distance from the axis of rotation
s = downstream coordinate, dimensionless
ω = rotational speed of the coordinate system
P = pressure side
S = suction side

Theme

IN radial compressor flow one usually can observe a flowfield at the impeller tip, which separates into a jet area and a wake area. As the wake area is attached to the suction side, it is assumed that separation occurs first on the suction side. This phenomenon was discussed experimentally and theoretically in an early paper by G. Jungclaus[1] for the case of the boundary-layer flow along a rotating, radial standing flat plate. He found that coriolis acceleration induces separation on the suction side earlier than on the pressure side. In the present paper, a solution is given for the laminar boundary layer on far more complicated blade surfaces as present for instance in high-speed turbomachinery, especially in radial compressors.

First, we outline the sort of simplifications which can be introduced in the general fluid mechanical equations to make a boundary-layer concept practicable for machine computation, while still providing reliable information on the boundary-layer behavior in such flows. 1) Second-order boundary-layer theory has to be applied, which means curvature terms, and coriolis and centrifugal acceleration terms of higher order are to be included. 2) The boundary layer is laminar, which may be acceptable for preliminary studies. The turbulent boundary layer is more energetic and therefore should lead to separation further downstream. 3) The boundary layer is compressible, because of the high relative speeds and the high temperatures in turbomachinery. 4) The boundary layer is assumed to be two-dimensional along a curve in space on the blade surface neglecting cross flow. Although this assumption is very crucial with respect to the calculation of the separation behavior, it has been made in order to simplify the effort to study the influences of the curvature, coriolis, and centrifugal effects which are the essentials considered in this investigation. 5) Since the boundary layer is nonsimilar no similarity assumption has been made, so that the boundary-layer profiles can develop continuously from the stagnation to the separation point.

A further problem is to find a natural blade orientated orthogonal coordinate system, which is able to represent the near wall region more easily than a Cartesian coordinate system. For this new coordinate system, the governing equations have been developed following Ref. 2. Details are given in the backup paper.[3]

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The nonlinear, parabolic system of partial differential equations of second order[3] is integrated numerically using an implicit higher-order "Mehrstellen" finite-difference scheme. This scheme was first used for boundary-layer calculations by E. Krause.[4] The main advantage of this method over first-order difference schemes lies in the fact that a much smaller number of grid points is necessary for a given degree of accuracy. This is because the first and second derivatives are collocated at three points, and not just at one point, as is the practice in conventional second-order difference schemes.

Additionally, variable step sizes in normal direction were used. This permitted a finer distribution of grid points in regions near the wall.

The discretization in the downstream direction was carried out using a second-order Lagrange interpolation polynomial. The integration of the continuity equation and the normal momentum equation were performed by the Simpson formula modified for variable step size.

In Ref. 3, a comparison with the Jungclaus[1] results was carried out showing good agreement for the range of rotational speed where realistic physical results were expected. When disagreement was found it could be shown that the boundary-layer simplifications were violated, as described in the following computations.

Following assumption 4, the geometry of a line on a radial compressor blade was taken from a high-speed radial compressor, for which the inviscid velocity distribution in the middle of the channel is known. Because of the preliminary nature of these calculations, this velocity distribution was taken as a boundary condition for the pressure-side boundary-layer calculations as well as for the suction-side calculations. For the same velocity distribution, variations of the rotational speed of the radial compressor were carried out, although the level of the velocity distribution is dependent on the rotational speed. All those weaknesses were tolerated, since the objective was to find the bounds of the applicability of the present boundary-layer solution method.

The results of such a parameter study for adiabatic flow are shown in Fig. 1. The abbreviations P 100 and S 100 are taken for the computations of the boundary-layer quantities on the pressure side and on the suction side, respectively, for a rotational speed of the compressor at $\omega = 100$ rps. In this figure, the skin friction coefficient c_f is plotted against the streamwise coordinate axis s for various rotational speeds. It is seen that increasing rotational speed makes the separation point move downstream, and this is valid for the suction side as well as for the pressure side. In any case, separation always occurs earlier on the suction side than on the pressure side for each rotational speed. At a certain limit, the solution procedure no longer gives reasonable results: c_f starts to grow, instead of falling continously. Although the increase of the rotational speed for this test series was made linear, the location of the separation point has a nonlinear behavior. The reason for this may be found in the nonlinear behavior of the

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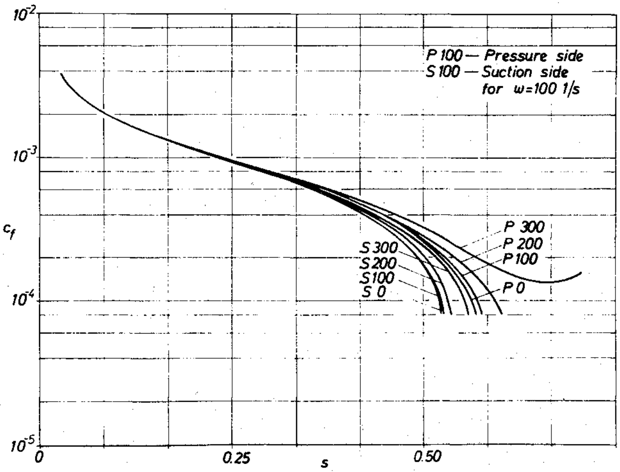

Fig. 1 Friction coefficient c_f along the downstream coordinate axis s in a radial compressor.

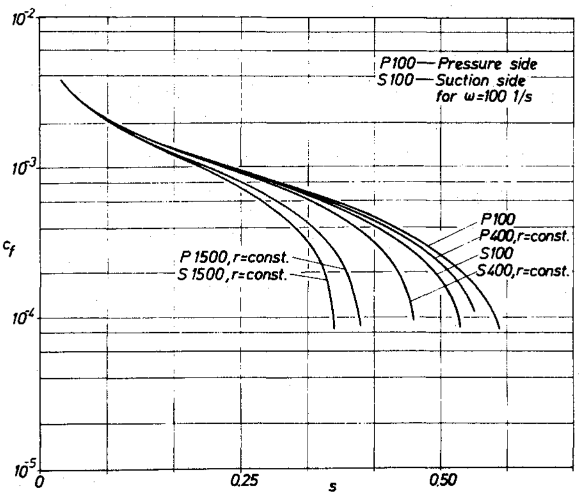

Fig. 2 Friction coefficient c_f along the downstream coordinate axis s in an axial compressor.

centrifugal acceleration term. So it might be concluded that the centrifugal and coriolis acceleration terms exeeded the limits required by the boundary-layer simplifications. This was found by comparing the order of magnitude of the theory inapplicable at high rotational speeds, it was of high convective and diffusion terms to that of the rotational and curvature terms.

Since it is conjectured that the additional acceleration terms caused by the rotation of the system make the boundary-layer interest to consider the axial compressor boundary layer too. This was done by using the same circumferential geometry of the radial compressor case, but the radius was held constant at the level of the radius at the inlet area. That means the radial curvature was neglected, and therefore the centrifugal acceleration was nearly constant over the whole blade length. In Fig. 2, the skin friction coefficient c_f is plotted against the streamwise coordinate axis s. Compared to the example S 100 and P 100 the separation point moves upstream when the rotational speed is increased. It is noted that, in this case of constant radius, realistic rotational speeds can be computed. The spatial radius of curvature is smaller than for the radial compressor; therefore the separation occurs earlier when the other parameters are constant.

From these results one may infer that a weak radial curvature has a positive effect on the location of the separation point, moving it downstream, but at the expense of high rotational speeds. One also may obtain some information about the applicability of boundary-layer theory in high-speed turbomachinery. The centrifugal acceleration terms are independent of the velocity components; they are dependent only on parameters like the geometry and the rotation. So pressure and centrifugal acceleration act partially like forcing functions. If they exceed in order of magnitude that of the remaining terms in the set of equations, they can change the character of the governing differential equations and make the solution scheme useless. To get information about the limiting order of magnitude of the acceleration due to rotation, especially about the centrifugal acceleration in non-Cartesian coordinate systems, further studies would be necessary, so that the range of validity of the boundary-layer theory for rotating systems could be defined more clearly.

References

[1] Jungclaus, G., "Grenzschichtuntersuchungen in rotierenden Kanälen bei scherenden Strömungen," Mitteilungen aus dem Max-Planck-Institut für Strömungsforschung, Göttingen, 11, 1955.

[2] Robert, K. and Grundmann, R., "Basic Equations for Non-Reacting Newtonian Fluids in Curvilinear Non-Orthogonal and Accelerated Coordinate Systems," DLR-FB 76-47, 1976.

[3] Grundmann, R., "Two-Dimensional, Laminar, Compressible Boundary Layer Calculations in Turbomachines," DLR-FB 76-38, 1976.

[4] Krause, E., "Mehrstellenverfahren zur Integration von Grenzschichtgleichungen," DLR-Mitt. 71-31, 1971.

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